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θ_{13} as a probe of $\mu \leftrightarrow \tau$ symmetry for leptons

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Abstract: Many experiments are being planned to measure the neutrino mixing parameter θ_{13} using reactor as well as accelerator neutrino beams. In this note, the theoretical significance of a high precision measurement of this parameter is discussed. It is emphasized that it will provide crucial information about different ways to understand the origin of large atmospheric neutrino mixing and move us closer towards determining the neutrino mass matrix. For instance if exact $\mu \leftrightarrow \tau$ symmetry in the neutrino mass matrix is assumed to be the reason for maximal $\nu_\mu - \nu_\tau$ mixing, one gets $\theta_{13} = 0$. Whether $\theta_{13} \simeq \sqrt{\Delta m^2_\odot/\Delta m^2_A}$ or $\theta_{13} \simeq \Delta m^2_\odot/\Delta m^2_A$ can provide information about the way the $\mu \leftrightarrow \tau$ symmetry breaking manifests in the case of normal hierarchy. We also discuss the same question for inverted hierarchy as well as possible gauge theories with this symmetry.

Keywords: Neutrino Physics, Beyond Standard Model.

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1. Introduction

Neutrino physics is poised on the brink of an exciting set of experiments that could elevate our knowledge of neutrino masses and mixings to the same level as that of quarks and charged leptons. At the same time, they are also likely to provide important information about physics beyond the standard model. The most crucial experiments in this regard are: (i) searches for neutrinoless double beta decay which will confirm whether neutrinos are Dirac or Majorana fermions; (ii) sign of the atmospheric mass difference which will determine whether the mass hierarchy is normal or inverted and (iii) the magnitude of the unknown angle θ_{13}, which will complete our knowledge of mixings.

In this article, I discuss the impact of a high precision search for θ_{13} assuming that neutrinos are Majorana fermions. There are several experimental proposals for such searches e.g. ref. [1, 2]. Some of these experiments are also likely to yield a more precise value of the atmospheric neutrino mixing angle. The value of θ_{13} in addition to providing a complete picture of neutrino mixings, could be a signal of the underlying physics responsible for lepton mixings and as such could be an important clue to physics beyond the standard model [3]. As is argued in this paper, value of θ_{13} in conjunction with a high precision measurement of the maximality of the atmospheric mixing angle $\theta_A \equiv \theta_{23}$ could indeed be a very useful way to determine the complete neutrino mass matrix for the case of a normal hierarchical spectrum for neutrinos.

To begin the discussion, let us note that the PNMS mixings arise from the lepton mass lagrangian as follows:

$$\mathcal{L}_m = \nu_\alpha^T C^{-1} \mathcal{M}_{\nu,\alpha\beta} \nu + \bar{e}_{\alpha,L} M^e_{\alpha\beta} e_R + \text{h.c.} \tag{1.1}$$

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Diagonalizing the mass matrices by the transformations $U_\nu^T \mathcal{M}_\nu U_\nu = \mathcal{M}^\nu_{\rm diag}$ and $U_\ell^\dagger M^e V = M^e_{\rm diag}$, one defines $U_{PMNS} = U_\ell^\dagger U_\nu$. Clearly, any symmetry in the lepton mass matrices is likely to manifest itself in the U_{PMNS} elements, at least in the basis where the charged leptons are mass eigenstates. We will parameterize U_{PMNS} as follows:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K, \tag{1.2}$$

where $K = {\rm diag}(1, e^{i\phi_1}, e^{i\phi_2})$.

To see how the symmetry of the mass matrix appears in the mixing matrix, let us consider the case of only two neutrino generations i.e. that of μ and τ. Experiments indicate that the atmospheric mixing angle is very nearly maximal i.e. $\theta_A = \pi/4$. Working in the basis where the charged lepton mass matrix is diagonal, it is obvious that the nautrino Majorana mass matrix that gives maximal mixing is:

$$\mathcal{M}_\nu^{(2)} = \begin{pmatrix} a & b \\ b & a \end{pmatrix}. \tag{1.3}$$

Furthermore the fact that solar neutrino mass difference square $\Delta m^2_\odot \ll \Delta m^2_A$ and allowing for small departures from the maximal atmospheric angle, we can write

$$\mathcal{M}_\nu^{(2)} = \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} 1+a\epsilon & 1 \\ 1 & 1+\epsilon \end{pmatrix}, \tag{1.4}$$

where a is a parameter of order one and $\epsilon \ll 1$. For the case of normal hierarchy we have $\sqrt{\Delta m^2_\odot/\Delta m^2_A} \simeq \frac{1}{4}(1+a)\epsilon$. The atmospheric mixing angle is given by $\theta_A \simeq \frac{\pi}{4} - \frac{\epsilon(1-a)}{4}$. It is clear if $a = 1$, the neutrino mass matrix has symmetry $\nu_\mu \leftrightarrow \nu_\tau$ and $\theta_A = \pi/4$. Thus departures from this symmetries remain imprinted in the values of the mixing angles.

2. Exact $\nu_\mu \leftrightarrow \nu_\tau$ symmetry and $\theta_{13} = 0$

Let us now extend the above considerations to the case of three generations. First point to note is that in the zeroth order, clearly unrealistic, approximation, maximal atmospheric mixing can arise from two kinds of neutrino mass matrices:

Case (i):

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \tag{2.1}$$

This is the case of normal hierarchy.

Case (ii):

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{2.2}$$

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This is the case of inverted hierarchy. Both these mass matrices are invariant under $\nu_\mu \leftrightarrow \nu_\tau$ symmetry. Furthermore, the second case has the additional symmetry : $L_e - L_\mu - L_\tau$ [4]. In both cases of course one has $\Delta m^2_\odot = 0$; $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. For the matrix in eq. (2.2), one also has in addition $\theta_{12} = \pi/4$.

In order to depart from this unrealistic zeroth order case to the more realistic case and to see how the various mixing angles are affected, let us first ask the question as to whether one can have mass matrices invariant under $\nu_\mu \leftrightarrow \nu_\tau$ symmetry while giving $\Delta m^2_\odot \neq 0$ and $\theta_{12} < \pi/4$. The answer to this question is "yes". An example of such a mass matrix is:

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_A}}{2} \begin{pmatrix} c\epsilon & d\epsilon & d\epsilon \\ d\epsilon & 1+\epsilon & -1 \\ d\epsilon & -1 & 1+\epsilon \end{pmatrix}. \tag{2.3}$$

Mass matrices of this type have been considered in [5]. A mass matrix with $\Delta m^2_\odot \neq 0$ but $\theta_{12} = \pi/4$ was discussed early on from considerations of $\nu_\mu \leftrightarrow \nu_\tau$ symmetry in [6]. Both these [7, 6] $\nu_\mu \leftrightarrow \nu_\tau$ symmetric neutrino mass matrices lead to $\theta_{13} = 0$.

For this mass matrix, we have

$$\begin{aligned} \epsilon &= 4\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \frac{1}{(c+1) + \sqrt{(c-1)^2 + 8d^2}} \\ \tan 2\theta_\odot &\simeq \frac{2\sqrt{2}d}{1-c} \\ \theta_{23} &= \frac{\pi}{4}; \theta_{13} = 0\,. \end{aligned} \tag{2.4}$$

Thus two of the three parameters of this matrix are determined by already existing data and if θ_{13} is found to be smaller than the limit expected in many forthcoming experiments and it is found that $\Delta m^2_{31} > 0$, then there would be a strong case for the matrix in eq. (2.3) as the mass matrix for the neutrinos (in the basis where the charged leptons are mass eigenstates) as well as for an underlying $\nu_\mu \leftrightarrow \nu_\tau$ symmetry. A test of this mass matrix would be a value of $\theta_{23} = \pi/4$. Since neutrinoless double beta decay can in principle determine the parameter c, one can determine all the parameters of this model. This would clearly be a major step forward in probing physics beyond the standard model.

We discuss the case of inverted mass hierarchy in subsequent section using the results in ref. [8].

3. Departures from $\nu_\mu \leftrightarrow \nu_\tau$ symmetry and expectations for θ_{13}

We now consider perturbations around the symmetric limit for the normal hierarchy case and discuss its consequences. Many discussions of such cases exist in the literature [9], (though not necessarily in the context of $\nu_\mu \leftrightarrow \nu_\tau$ symmetry). We motivate our discussion from the angle of this symmetry. We mostly discuss the case without CP violation and in the end of this section, comment on a case with CP violation.

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The most general CP conserving perturbation of the neutrino mass matrix around the $\nu_\mu \leftrightarrow \nu_\tau$ symmetric limit that maintains the hierarchy $\Delta m^2_\odot \ll \Delta m^2_A$ and near maximal atmospheric mixing is:

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_A}}{2}\begin{pmatrix} c\epsilon & d\epsilon & b\epsilon \\ d\epsilon & 1+a\epsilon & -1 \\ b\epsilon & -1 & 1+\epsilon \end{pmatrix}. \tag{3.1}$$

The parameters characterizing the departures from symmetry limit are: $b \neq d$ and $a \neq 1$. Two characteristic predictions appear depending on the way the symmetry breaking appears in the mass matrix.

Case (i): $a = 1, b \neq d$**.** In this case, we diagonalize the mass matrix for the case when $c \ll 1$ and ignoring terms of order $(b-d)/(b+d)$ in ϵ but keeping them in θ_{13}. (Keeping these terms in ϵ gives a somewhat complicated expression and since we are interested in qualitative predictions, we do not include these corrections). We find that

$$\begin{aligned} \epsilon &\simeq \frac{4}{\sqrt{1+8d^2}}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \\ \theta_{13} &\simeq (b-d)\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \\ \tan 2\theta_\odot &\simeq \frac{2(b+d)}{1-c} \\ m_{\beta\beta} &\simeq c\epsilon\,. \end{aligned} \tag{3.2}$$

Using present data, in this case one would expect θ_{13} slightly below its present upper limit (say around 0.15 or so). The predictions in models where atmospheric neutrino mixing arises from some dynamical mechanism [10] are also similar. The difference between this approximate $\mu \leftrightarrow \tau$ symmetry case and the "dynamical" case is that the atmospheric mixing angle in the symmetry case being discussed here is very close to maximal with departure from maximality being of order $\frac{\Delta m^2_\odot}{\Delta m^2_A}$ which is a few per cent (of order $\leq 4^0$) whereas in the dynamical case, this departure can be larger (of order $\sim 8^0$ or so). The prediction for neutrinoless double beta decay in this case is beyond the range of accessibility of the next round of searches for double beta decay [11].

The physical meaning of this case is that while $\nu_\mu \leftrightarrow \nu_\tau$ symmetry is exact in the $\nu_\mu - \nu_\tau$ sector, it is broken in their mixing with ν_e. We will call this e-sector breaking. Unless this breaking is constrained by extra symmetries, one would expect a large θ_{13}, as noted.

Case (ii): $a \neq 1, b = d$**.** In this case, we get

$$\begin{aligned} \epsilon &\simeq \frac{4}{[c+(1+a)/2]+\sqrt{[c-(1+a)/2]^2+8d^2}}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \\ \theta_{13} &\simeq \frac{1}{4\sqrt{2}}\epsilon^2 d(1-a)\,. \end{aligned}$$

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symmetry breaking	θ_{13}	$\theta_{23} - \pi/4$
None	0	0
$\mu - \tau$ sector only	$\sim \Delta m^2_{\odot}/\Delta m^2_A$	$\leq 8^0$
e-sector only	$\sim \sqrt{\Delta m^2_{\odot}/\Delta m^2_A}$	$\leq 4^0$
dynamical	$\sim \sqrt{\Delta m^2_{\odot}/\Delta m^2_A}$	$\leq 8^0$

Table 1: This table gives the predictions for θ_{13} and θ_A for different ways of $\mu \leftrightarrow \tau$ symmetry breaking. Note that what we mean by e-sector only is that $\mu \leftrightarrow \tau$ symmetry is broken in the $e - \mu$ and $e - \tau$ elements of the Majorana neutrino mass matrix. Similarly, when we say $\mu - \tau$ sector, we mean the symmetry is broken in $\mu - \tau$ subsector of the neutrino mass matrix.

In this case there is a departure from maximality of the atmospheric mixing angle given by the following equation:

$$\theta_A \simeq \frac{\pi}{4} - \epsilon\frac{1-a}{4} \tag{3.3}$$

Thus, the expectation for θ_{13} for this way of symmetry breaking is around $\theta_{13} \approx 0.03$. The smallness of θ_{13} here compared to the previous case can be understood as follows: the $\nu_\mu \leftrightarrow \nu_\tau$ symmetry is broken in the only in the $\nu_\mu - \nu_\tau$ sector of the mass matrix and not in the mixing with ν_e. As a result, to leading order in $\epsilon \approx \sqrt{\frac{\Delta m^2_{\odot}}{\Delta m^2_A}}$, there is no contribution to θ_{13} and it arises only to order ϵ^2. Also as noted above, the departure from maximality of the atmospheric mixing angle in this case can be significant ($\sim 8-10\%$).

Case (iii): $a = 1; |b| = |d|$. An interesting way to break $\nu_\mu \leftrightarrow \nu_\tau$ is to maintain $a = 1$ so that symmetry breaking is in the mixing with ν_e; but choose $b = d^*$[14]. In this case, one has $\theta_{13} = 2Imb\sqrt{\frac{\Delta m^2_{\odot}}{\Delta m^2_A}}$ and one has the Dirac phase at its maximal value of $\pi/2$.

In the table 1, we summarize our results.

4. Departures from $\nu_\mu \leftrightarrow \nu_\tau$ symmetry: the inverted hierarchy case

The case of inverted hierarchy has been discussed in great detail in [8] (although connection to $\mu \leftrightarrow \tau$ symmetry was not discussed). Here I summarize the discussion in the language of $\mu \leftrightarrow \tau$ symmetry.

The most general mass matrix in this case is:

$$\mathcal{M}_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} z & c & s \\ c & y & d \\ s & d & x \end{pmatrix}, \tag{4.1}$$

where c and s stand for *cos* and *sin* of θ_{23} and $x, y, z, d \ll 1$. In the perturbative approximation, we find the following sumrules involving the neutrino observables and the elements

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of the neutrino mass matrix. It follows from this matrix that

$$
\begin{aligned}
\sin^2 2\theta_\odot &= 1 - \left(\frac{\Delta m^2_\odot}{4\Delta m^2_A} - z\right)^2 + O(\delta^3) \\
\frac{\Delta m^2_\odot}{\Delta m^2_A} &= 2(z + \vec{v}\cdot\vec{x}) + O(\delta^2) \\
U_{e3} &= \vec{A}\cdot(\vec{v}\times\vec{x}) + O(\delta^3)\,,
\end{aligned}
\tag{4.2}
$$

where $\vec{v} = (\cos^2\theta, \sin^2\theta, \sqrt{2}\sin\theta\cos\theta)$, $\vec{x} = (x, y, \sqrt{2}d)$ and $\vec{A} = \frac{1}{\sqrt{2}}(1,1,0)$. δ. Now we can discuss the exact $\nu_\mu \leftrightarrow \nu_\tau$ limit and departures from it. The exact symmetry limit occurs when we have $c = s = \frac{1}{\sqrt{2}}$ (maximal atmospheric mixing angle) and $x = y$. It is clear from above that $\theta_{13} = 0$ in this limit. Therefore, a nonvanishing θ_{13} is related to breakdown of this symmetry as in the case of normal hierarchy.

It is clear from this way of parameterizing the mass matrix that the current best fits for the large mixing angle solution to the solar neutrino observations[12] require $z \geq 0.3$ or so. This translates into a lower limit on $m_{\beta\beta} \geq 15\,\mathrm{MeV}$ [13]. Similar to the case of normal hierarchy case, there are two broken symmetry situations.

Case (i): $c = s = \frac{1}{\sqrt{2}}$; $x \neq y$. In this case, we have

$$
\begin{aligned}
\theta_{13} &= \frac{x-y}{2} \\
\frac{\Delta m^2_\odot}{\Delta m^2_A} &= 2(x + y + z + d)\,.
\end{aligned}
\tag{4.3}
$$

In this case, θ_{13} could be quite large. It is worth noting that in this case even though smallness of $\frac{\Delta m^2_\odot}{\Delta m^2_A}$ implies that there must be cancellations among the parameters x, y, z and d, it does not put any constraint on how large θ_{13} can be.

Case (ii): $c \neq s$; $x = y$. In this case we find

$$
\theta_{13} \simeq -d\cos 2\theta_A\,. \tag{4.4}
$$

In this case, there is a close connection between the value of θ_{13} and departure from maximality of θ_A.

It is clear that the expectations for θ_{13} for the inverted hierarchy are very different from the normal hierarchy case. Specially missing in this case is the close connection between θ_{13} and the ratio $\frac{\Delta m^2_\odot}{\Delta m^2_A}$. The reason for this is that the value of $sin^2 2\theta_\odot$ required by the present solar and KamLand data requires the m_{ee} term in the neutrino mass matrix to be large in the case of inverted hierarchy. This therefore enters as a new parameter in the $\Delta m^2_\odot$ unlike the case of normal hierarchy.

5. Possible gauge theory of broken $\nu_\mu \leftrightarrow \nu_\tau$

So far the discussion has focussed on the testability of $\nu_\mu \leftrightarrow \nu_\tau$ symmetry in the neutrino Majorana mass matrix. In this section we would like to address its implications for physics

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beyond the standard model. We would like to seek plausible gauge models that lead to this symmetry. We will focus only on the normal hierarchy case since the case of inverted hierarchy has been studied extensively in the literature [4].

The first clear obstacle one must overcome is that the neutrinos are part of the $\mathrm{SU}(2)_L$ doublet that contains the charged leptons (e, μ, τ) and there is no apparent $\mu \leftrightarrow \tau$ symmetry in the charged lepton masses. However, in the limit of $m_\mu = m_\tau$, one can have such a symmetry implying that in the charged lepton sector, there must clearly be a mechanism to break the symmetry by a large amount without affecting the neutrino. We explore below how such a symmetry can emerge in gauge theories and in particular, how it can be broken in a consistent manner. The goal is a modest one of simply trying to give an existence proof. The main point is that if one cannot even construct a consistent model within the loose framework of arbitrary fine tuning, the symmetry has a less chance of being meaningful in reality. In our case it turns out that in addition to assuming a spontaneously broken $\mu \leftrightarrow \tau$ symmetry, if one assumes a Z_4 symmetry, then there are several models that one can construct that realize the mass matrix in eq. (2.3) without conflicting with charged lepton spectrum with rather mild assumptions. We only discuss the symmetric limit. One can easily extend them to include small breaking effects e.g. by adding higher dimensional terms to the lagrangian.

We first show that for θ_{13} to vanish, the $\mu \leftrightarrow \tau$ symmetry must be in the left handed neutrino sector; in other words, if we had the permutation symmetry only in the RH neutrino sector, it does not lead to a vanishing θ_{13}. We then present two models one with right handed neutrinos and one without them where $\mu \leftrightarrow \tau$ symmetry is imposed both in the left and right handed sector and show that it leads to vanishing θ_{13} in the symmetry limit. In the first case we will use the conventional seesaw mechanism and in the second one, we will use a triplet dominated type II seesaw [15].

5.1 Model I

We now consider models where $\mu \leftrightarrow \tau$ applies both in the left and right handed neutrino sector. We use the standard model gauge group with supersymmetry and standard assignment of matter superfields [16] but with three pairs of Higgs doublets (H_u, H_d). We impose on the model an $S_2 \times Z_4$ symmetry. The multiplets (L_μ, L_τ), (μ^c, τ^c), (N^c_μ, N^c_τ), $(H_{d,1}, H_{d,2})$ transform into each other under S_2 symmetry and $H_{u,i}$ (i=1,2,3) and the rest of the fields transform as singlets. Under Z_4, we assign $(\mu^c, H_{d,2})$ to transform as $i(\mu^c, H_{d,2})$ whereas $(\tau^c, H_{d,1})$ go to $-i(\tau^c, H_{d,1})$. Rest of the fields are invariant. First point to note is that, the right handed neutrino mass matrix invariant under this has the form

$$M_R = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix}. \tag{5.1}$$

The Dirac mass matrix for the neutrinos also has similar form:

$$m_D = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}. \tag{5.2}$$

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It is clear that the neutrino mass matrix obtained from the above two equations after type I seesaw has the form which is as in eq. (2.3) which is $\mu \leftrightarrow \tau$ invariant.

To complete the discussion of model I, note that the charged lepton mass matrices arise from the superpotential:

$$W = h_1(L_\mu H_{d,1}\mu^c + L_\tau H_{d.2}\tau^c) + h_e L_e H_{d,3} e^c + h_3(L_\tau H_{d1}\mu^c + L_\mu H_{d2}\tau^c)\,. \tag{5.3}$$

Now if we set $h_3 = 0$ and suppose that we break the $\mu \leftrightarrow \tau$ symmetry by the soft $H_{d,1,2}$ mass terms, then $H_{d,1,2}$ will have different and arbitrary vevs. As a result, we can get correct values for all the charged lepton masses.

5.2 Model II without right handed neutrinos

This model is very similar to the model above except that there are no right handed neutrinos- instead there are Higgs triplets Δ_L with standard model hypercharge $+2$ so that couplings of type $LL\Delta_L$ are allowed. The Δ_L is given a mass term M which is of order of the 10^{14} GeV, so that the vev of Δ_L is suppressed due to the term $\Delta_L H_d H_d$ to be $v_{wk}^2/M \simeq 10^{-1}$ eV, which can give neutrino masses of the right order. As in the first case, we require the model to be invariant under $S_2 \times Z_4$ symmetry with assignments as in the previous case. The fields in $\Delta_L \oplus \bar{\Delta}_L$ pair are invariant under it. The neutrino masses come from the superpotential

$$f_1(L_\mu + L_\tau)((L_\mu + L_\tau))\Delta_L + f_2(L_\mu - L_\tau)((L_\mu - L_\tau))\Delta_L + (L_\mu + L_\tau)L_e\Delta_L + L_e L_e \Delta_L\,. \tag{5.4}$$

Again this leads to a neutrino mass matrix invariant under $\mu \leftrightarrow \tau$ symmetry. The charged lepton masses arise in exactly the same way as in the model I.

There are also other models in the literature with similar properties (e.g. see ref. [17]) also. It would therefore seem that considering $\mu \leftrightarrow \tau$ symmetry for leptons, despite its strong breaking in the charged lepton sector is quite a meaningful and useful way to obtain information about physics beyond the standard model from neutrinos.

Let us make a few comments on the models described above. Note that we have not incorporated any breaking of $\mu \leftrightarrow \tau$ into the model. There could many many sources for such breakings: for example, there could be higher dimensional operators that involve $H_{d,1,2}$ that can break this symmetry. There could also be other effects such as radiative corrections from charged lepton Yukawa couplings that give mass to tau lepton and the muon etc.

6. Summary and conclusion

In this brief note, it is pointed out that the measurement of the neutrino mixing angle θ_{13} in conjunction with a measurement of the departure from maximality of the atmospheric mixing angle can be a very powerful way to probe any possible $\nu_\mu \leftrightarrow \nu_\tau$ symmetry present in the neutrino mass matrix. In Table I, the expectations for θ_{13} and different cases (with and without approximate $\nu_\mu \leftrightarrow \nu_\tau$ symmetry) are presented for the case of normal hierarchy and can be used as a way to specify the mass matrix. We also have discussed the case of inverted mass hierarchy and pointed out the implications of broken $\mu \leftrightarrow \tau$ symmetry.

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Evidence for any approximate $\nu_\mu \leftrightarrow \nu_\tau$ symmetry will clearly be a significant indicator of which way to proceed as we probe physics beyond the standard model. For instance, such a symmetry is highly nontrivial to obtain within the framework of grand unification and pint to alternative directions, which will be a useful information.

We must emphasize that all our considerations are based on the assumption that there are no extra sterile neutrinos mixing with the three known active ones. Claerly therefore any evidence for sterile neutrinos will require a re-evaluation of the conclusions stated in the paper.

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